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ADDENDUM

Random sequential packing fraction in square cellular structures

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Abstract. Hitherto the problem of random sequential packing has been treated in continuum space. In a previous paper we introduced random sequential packing into the square cellular structure, where squares with integer length are inserted at random without any overlap into the cells of a square substrate divided into square unit cells. To the packing we applied two methods A and B, which were not distinguished in the packing of continuum space. In A, contact between the packed squares is permitted, and in B such contact is forbidden. In the previous paper we independently obtained the packing fractions of A and B, and did not notice the relation between them. Here, however, we make clear the relation between the packing fractions of A and B and continuum space.

The problem of random sequential packing (RSP) is mathematically and physically interesting, and has been studied in many fields. Hitherto the problem has been treated mainly in continuum space. In our previous paper (Nakamura 1986), we introduced the problem into the square cellular structure, where squares with integer length a are inserted at random one by one without any overlap into the cells of a square substrate divided into square unit cells. Then the sides of the inserted squares are just put on the cell boundaries of the substrate.

In such packing problems, two methods A and B, which are not distinguished in the RSP of continuum space, are applied to filling squares. In A, contact between the filled squares is permitted, and in B contact is forbidden. In the previous paper, we independently calculated the packing fractions of A and B against a, using a computer simulation, and did not notice the relation between them. Here we make clear the relation between the packing fractions of A and B and continuum space.

We present an example of the packing pattern using method B for $a = 1$ in figure 1. We elongate the lengths of all the filled squares by one in the right and lower directions and in figure 2 we depict the pattern after this operation. The shaded parts are stretched areas, and we can see that the texture of figure 2 is a packing texture obtained by using method A for $a = 2$. Thus we derive the relation between the packing fractions as

$$p_A(2) = 4p_B(1). \tag{1}$$

More generally

$$p_A(a+1) = [(a+1)^2/a^2]p_B(a) = (1+1/a)^2 p_B(a) \qquad (a \geq 1) \tag{2}$$

where $p_A(a)$ and $p_B(a)$ are the packing fractions of A and B respectively when the lengths of the filled squares are a. We can obtain a packing pattern of A by making the lengths of all the filled squares in a pattern of B longer by one. We show relation (2) in table 1 for the values obtained by computer simulation in our previous paper. As indicated in table 1, relation (2) is well fitted to the values from computer simulation.

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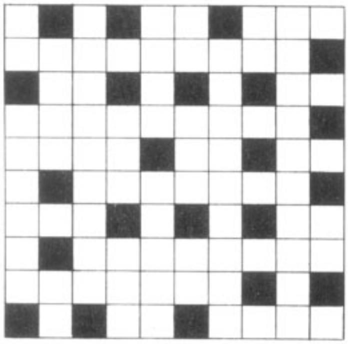

Figure 1. A random sequential packing pattern obtained by method B for $a = 1$.

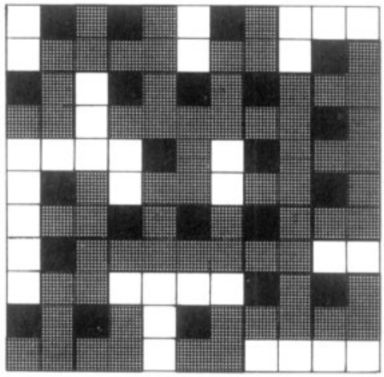

Figure 2. A random sequential packing pattern obtained by method A for $a = 2$ formed from figure 1.

Table 1. Packing fractions.

	a	1	2	3	4	5	6
Packing	$p_B(a)$	0.187	0.302	0.364	0.404	0.429	0.447
fraction	$p_A(a)$	1.000	0.749	0.681	0.646	0.628	0.620
	$[(a+1)^2/a^2]p_B(a)$	—	0.748	0.680	0.647	0.631	0.618

Many articles have been published on the problem of the RSP of orientated squares in the continuum plane (Akeda and Hori 1976, Feder 1980, Finegold and Donnell 1979, Jodrey and Tory 1980, Palasti 1960). When we denote by p_C the packing fraction of the continuum plane, the relation between p_A, p_B and p_C is

$$p_A(a) > p_C > p_B(a). \tag{3}$$

In the limit of $a \to \infty$, the relation

$$\lim_{a\to\infty} p_A(a) = \lim_{a\to\infty} p_B(a) \tag{4}$$

is derived from (2). Therefore from (3) and (4) the relation between p_A, p_B and p_C is derived as follows

$$p_A(\infty) = p_C = p_B(\infty). \tag{5}$$

However, the speed of convergence of p_A to p_B at $a \to \infty$ is very slow because of

$$(1 + 1/a)^2 \simeq 1 + 2/a. \tag{6}$$

The slowness was also shown in our previous paper. Because of the slowness of convergence, the problem of RSP in the square cellular structure must by itself be significant, and it is inefficient to execute a computer simulation in cellular structures in order to obtain approximate packing fractions for continuum space.

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